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A NEW HYBRID MODE BOUNDARY INTEGRAL METHOD FOR ANALYSIS OF MMIC WAVEGUIDES WITH COMPLICATED CROSSECTION

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ABSTRACT

A boundary integral formulation for quasistatic, TE, TM and hybrid wave analysis of open or shielded waveguides with arbitrary multiregion crossection is presented. A special form of boundary integral equation is derived first to make possible the numerical treatment of cornered geometries. Subsequently operator equations including source terms are given for analysis of arbitrary 2-D structures. The numerical method is described shortly, including as example the quasistatic analysis of coplanar waveguide with non-rectangular conductor shape.

INTRODUCTION

A variety of complicated crossections is exhibited by active and passive MMIC waveguiding structures which on principle or by technology are neither planar nor of rectangular geometry, therefore excluding some standard methods for their analysis. To deal with arbitrary multiregion crossections the flexibility of the finite element method (FEM), the finite difference method (FDM) or the boundary integral equation method (BIEM) is required. While the former have been applied to waveguides by different workers (1), application of the BIEM has only been reported for quasistatic problems (2) and no BIE formulation for the analysis of hybrid wave propagation has yet been given.

With the BIEM (3,4) a partial differential equation for an unknown function defined over some domain is transformed into an integral equation for its boundary values. The principal advantage of this approach is the reduction of problem dimensionality by one which directly translates into reduced matrix size and facilitates data preparation. As opposed to the domain methods unbounded regions present no difficulties to the BIEM hence no artificial shielding is required. For multiregion problems interface continuity conditions are automatically satisfied by stating the BIE for each subdomain in terms of those field components that are continous over interfaces. As solution of the BIE system the interface values of these field components are obtained as the primary result. Integral quantities and field components at interior points are easily computed from them by one-dimensional integration. Due to the one-dimensionality of the transformed problem a very flexible numerical implementation is possible, including for example automatic generation of the asymptotically exact edgeterms for metallic and dielectric corners, such permitting to obtain the precise fields in the vincinity of corners while further reducing the number of unknowns that are needed. Curved boundaries do not need polygonal approximation but can be entered in their exact functional form.

The method has a wide range of applications. For static analysis anisotropic media can be handled by simply changing the fundamental solution subroutine, also nonlinear conductivity may be specified. The hybrid wave formulation allows for regions of finite conductivity so as to model losses and slow wave effect. We have included source terms for both the static and the full wave BIE system thus aiming at future extension of the model to the analysis of active structures.

BOUNDARY INTEGRAL FORMULATION

This section first introduces a modified *implicit* form of BIE on which the present method is based. This form is equally applicable to smooth and cornered boundaries while the standard form fails in the latter case, due to loss of uniform convergence of some integral operator.

To start with, consider an open, unbounded or bounded domain $\Omega \subset \mathbb{R}^2$, for the moment assume its boundary $\partial\Omega$ to be smooth and let $\boldsymbol{n}$ denote its outer normal. In Ω a solution is sought for the problem

$$\mathrm{L}u := \Delta u + h^2 u = -b \qquad (1)$$

with $h^2 \in \mathbb{C}$ and u subject to some boundary condition $\mathrm{B}(u, \boldsymbol{n}\,\mathrm{grad}\,u) = 0$ on $\partial\Omega$. The space of admissible functions u is restricted by requiring that u be twice continously differentiable in the open domain Ω and Hölder continous on its closure $\overline{\Omega}$, i.e.

$$|u(\boldsymbol{p}) - u(\boldsymbol{q})| \le M \,\|\boldsymbol{p} - \boldsymbol{q}\|^{\tau} \qquad (2)$$

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with $M, \tau \in \mathbb{R}_+$ for all $p, q \in \overline{\Omega}$, the necessity of the latter restriction becoming obvious below. For brevity we introduce the symbol

$$v : \partial\Omega \to \mathbb{C},\ q \mapsto n(q) \operatorname{grad} u(q)$$

for the normal derivative of u on $\partial\Omega$. Translating the boundary value problem into a boundary integral formulation employs a *fundamental solution*

$$g(p,\cdot) : \overline{\Omega} \setminus \{p\} \to \mathbb{C},\ (p,q) \mapsto g(p,q)$$

of eq.(1) which for any $p \in \overline{\Omega}$ it satisfies

$$\mathbf{L}_q\, g = -A(p)\, \delta_2(\|p-q\|).$$

$A(p)$ here denotes the plane angle that opens from p into Ω and δ_2 is Diracs distribution in $\mathbb{R}^2$. The fundamental solution is not unique. It depends on the parameter h and in addition may have the symmetries of the problem built into it, but always can be expressed as sum of its singular part

$$g_0(p,q) := -\ln(\|p-q\|) \tag{3}$$

and a function g_1 which is defined and continuously differentiable on an open domain $U \supset \overline{\Omega}$. Consequently its normal derivative on $\partial\Omega$

$$k(p,\cdot) : \partial\Omega \setminus \{p\} \to \mathbb{C},\ (p,q) \mapsto n(q) \operatorname{grad}_q g(p,q)$$

may be expressed as sum of

$$k_0(p,q) = \frac{(p-q)\,n(q)}{\|p-q\|^2} \tag{4}$$

and a continous function k_1. Application of Greens second theorem to the functions u and g with respect to the punctured domain $\Omega \setminus \{p\}$ renders the BIE

$$A(p)u(p) + \int_{\partial\Omega\setminus\{p\}} k(p,q)\, u(q)\, ds(q) - \int_{\partial\Omega\setminus\{p\}} g(p,q)\, v(q)\, ds(q) = \iint_{\Omega\setminus\{p\}} g(p,q)\, b(q)\, d^2q \tag{5}$$

in its standard *explicit* form. This form is well suited to regions with a smooth boundary $\partial\Omega$ where the appearing boundary integrals converge uniformly. To see this only the singular parts g_0, k_0 of the kernels need consideration. The assumption of a smooth boundary says that for any two points $p, q \in \partial\Omega$ and some $C \in \mathbb{R}_+$ the inequality

$$|(p-q)\,n(q)| \le C\, \|p-q\|^2 \tag{6}$$

holds as $q \to p$. Hence we have

$$|k_0(p,q)\, u(q)| \le C \sup_{q \in \partial\Omega} (|u(q)|) \tag{7}$$

and the first boundary integral on the left hand side of (5) is regular in this case. The second integral is weakly singular but uniform convergence is assured by Lebesgue's dominated convergence theorem as

$$|g_0(p,q)\, v(q)| \le |\ln(\|p-q\|)|\ M\, \|p-q\|^{\tau-1}$$

because of (2). If now the assumption of $\partial\Omega$ beeing smooth is dropped and singular boundary points are admitted as depicted in Fig.1, obviously inequalities (6) and (7) no longer hold and uniform convergence of the first boundary integral in eq.(5) is lost. Discretization of the BIE by projecting it onto a set of test-functions would however require that this integral again appeared under an integral. As both integrals are to be evaluated numerically, we conclude that eq.(5) is not applicable in the presence of singular boundary points. To overcome this problem eq.(5) is replaced by

$$\int_{\partial\Omega\setminus\{p\}} k_0(p,q)\big(u(q)-u(p)\big) ds(q) + \int_{\partial\Omega\setminus\{p\}} k_1(p,q)\, u(q)\, ds(q) - \int_{\partial\Omega\setminus\{p\}} g(p,q)\, v(q)\, ds(q) = \iint_{\Omega\setminus\{p\}} g(p,q)\, b(q)\, d^2q, \tag{8}$$

the *implicit form* of the BIE. For smooth boundaries equations (5) and (8) are equivalent. The implicit form however does not suffer from loss of uniform convergence of the boundary integrals in the presence of singular boundary points. This follows from the dominated convergence theorem again, for

$$\left|k_0(p,q)\big(u(q)-u(p)\big)\right| \le \frac{1}{\|p-q\|}\ M\, \|p-q\|^{\delta}$$

as $q \to p$ as consequence of inequality (2).

Eq.(8) is the basis of the present method. Before proceeding to the applications we rewrite it in operator notation as

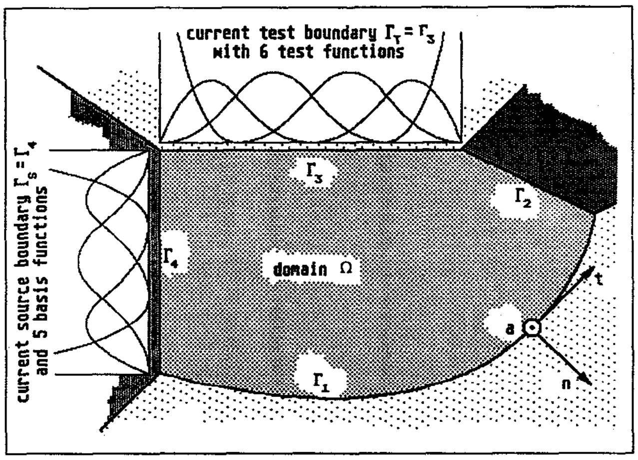

Fig.1: A typical domain Ω of a general 2-D structure with non-smooth boundary $\partial\Omega$ made up of the smooth curves $\Gamma_1 \ldots \Gamma_4$ which join in singular boundary points. A local right handed system of unit vectors (n, t, a) is attached to each boundary point. Basis and test functions are shown for two boundaries, the current source and test boundary (see text).

$$\mathbf{K}[u](p) - \mathbf{G}[v](p) = \mathbf{Q}[b](p), \tag{9}$$

where the definition of the operators should be obvious by comparison, and shortly indicate how discretization of the operators is performed. Pairs of curves $\Gamma_T, \Gamma_S \in \{\Gamma_i\} \subset \partial\Omega$ are selected in succession to serve as *test-* and *source-boundary* respectively (Fig.1). Having chosen suitable sets of *test functions* $\{f_m\}: \Gamma_T \to \mathbb{R}$ and *basis functions* $\{u_n\}, \{v_n\}: \Gamma_S \to \mathbb{R}$ partial discrete operators of the form

$$\left(\mathbf{G}_{mn}\right)_{TS} := \left(\int_{\Gamma_T} f_m(t) \int_{\Gamma_S \setminus \{p(t)\}} g(p(t), q(s))\, v_n(s)\, ds\, dt \right)$$

can be computed and finally be assembled to the global matrix equation. An iterative numerical integration scheme is used to compute the operators up to a predefined relative precision. The important questions of how to select regular and singular basis functions (splines and inverse fractional power) for a given structure and how to regularize the quasisingular integrals can not be described in detail here but some information is interspersed below.

STATIC FIELDS

The static field problem was tackled first to work out the details of the method and also for its practical relevance, as many typical transmission line structures encountered in MMICs have transverse dimensions small enough to justify the quasistatic approximation. Solving for the electrostatic potential is a straightforward application of eq.(9) with h^2 set to zero. Each homogenous subregion contributes a BIE

$$\mathbf{K}[\varphi] + \frac{1}{\varepsilon}\mathbf{G}[\boldsymbol{n}\cdot\boldsymbol{D}] = \frac{1}{\varepsilon}\mathbf{Q}[\rho] \tag{10}$$

where φ denotes electrostatic potential and $\boldsymbol{n}\cdot\boldsymbol{D}$ the normal component of electric displacement. The domain integral operator $\mathbf{Q}$ only appears when domain charge density ρ is present. As φ and $\boldsymbol{n}\cdot\boldsymbol{D}$ will be known on some boundaries and unknown on others eq.(10) must be rearranged correspondingly. For simplicity we indicate this here by writing

$$\left(\mathbf{K}[\varphi] + \frac{1}{\varepsilon}\mathbf{G}[\boldsymbol{n}\cdot\boldsymbol{D}]\right)_{\text{unknown}} = \frac{1}{\varepsilon}\mathbf{Q}[\rho] - \left(\mathbf{K}[\varphi] + \frac{1}{\varepsilon}\mathbf{G}[\boldsymbol{n}\cdot\boldsymbol{D}]\right)_{\text{known}}.$$

After computing the partial operators for all combinations of test and source boundaries in each domain, assembly of the global operator equation merely requires the block matrices to be put in their right places and their sign adjusted according to wether the source boundary normal points out of or into the current domain. Note that interface continuity conditions are automatically fulfilled by selecting φ and $\boldsymbol{n}\cdot\boldsymbol{D}$ as the boundary value functions to work with.

As demonstration a coplanar waveguide with nonrectangular conductor shape as due to underetching (Fig.2a) and electrolytical growth (Fig.2b) respectively is analyzed. It was found that the technological impact on conductor shape is by no means negligible in MMIC applications (Fig.3). Surface charge density and interface flux (Fig.4) are computed to high precision using only few basis functions because edgeterms with asymptotically correct order of singularity are included as was indicated in Fig.2. The program automatically determines the appropriate exponents for each corner by first solving Laplace's equation for a sectorial medium. Apart from the ground plane a vertical symmetry plane was specified in the program input for this example, causing it to construct the regular part of the fundamental solution by superimposing images of g_0 and restrict computation to one half of the structure.

HYBRID WAVE BIE-SYSTEM

This section generalizes the above BIE approach to full wave analysis. To this end the material in each homogenous subregion Ω is described by means of the complex parameters $z := j\omega\mu_0\mu_r$ and $y := \sigma + j\omega\varepsilon_0\varepsilon_r$. For the electromagnetic field $\boldsymbol{E}, \boldsymbol{H}$ we assume $\exp(j\omega t - \gamma \boldsymbol{a}\boldsymbol{r})$ behaviour. In addition to $\sigma\boldsymbol{E}$ an 'independent' domain current density $\boldsymbol{S}_e$ with the same propagation factor is taken into account in the derivation to prepare for future extensions of the model to active structures. Restricting ourselfes to solenoidal fields here and describing the 'independent' current density $\boldsymbol{S}_e$ by

$$\boldsymbol{S}_e := \operatorname{rot} \boldsymbol{a}\eta + \operatorname{rot}\operatorname{rot} \boldsymbol{a}\vartheta$$

with $\eta, \vartheta: \Omega \to \mathbb{C}$, the electromagnetic field is derived from two scalar potentials $\chi, \psi: \Omega \to \mathbb{C}$ in the form

$$\boldsymbol{E} = z\,\boldsymbol{a}\times\operatorname{grad}\chi - \gamma\operatorname{grad}\psi + h^2\boldsymbol{a}\psi - z\boldsymbol{a}\vartheta$$

and

$$\boldsymbol{H} = -\gamma\operatorname{grad}\chi + h^2\boldsymbol{a}\chi - y\,\boldsymbol{a}\times\operatorname{grad}\psi + \boldsymbol{a}\eta - \boldsymbol{a}\times\operatorname{grad}\vartheta\,,$$

where $h^2 := \gamma^2 - zy$ and χ and ψ are solutions of $\mathbf{L}\chi = -\eta$ and $\mathbf{L}\psi = z\vartheta$, respectively. Expressing the potentials χ and ψ and their normal derivatives by the proper components of the electromagnetic field the latter two equations are translated into the coupled system

$$\mathbf{K}[\boldsymbol{aH}] - \frac{h^2}{z}\mathbf{G}[\boldsymbol{tE}] - \frac{\gamma}{z}\mathbf{G}[\tfrac{\partial}{\partial s}\boldsymbol{aE}] = -\mathbf{Q}[\operatorname{div}\boldsymbol{a}\times\boldsymbol{S}_e] + \mathbf{G}[\boldsymbol{t}\boldsymbol{S}_e],$$

$$\mathbf{K}[\boldsymbol{aE}] + \frac{h^2}{y}\mathbf{G}[\boldsymbol{tH}] + \frac{\gamma}{y}\mathbf{G}[\tfrac{\partial}{\partial s}\boldsymbol{aH}] = -z\,\mathbf{Q}[\boldsymbol{a}\boldsymbol{S}_e] + \frac{\gamma}{y}\mathbf{G}[\boldsymbol{n}\boldsymbol{S}_e] \tag{11,12}$$

of a magnetic (MFIE) and an electric field integral equation (EFIE). Interface continuity conditions again are automatically fulfilled with this formulation. In the special case of pure TE or TM propagation and zero $\boldsymbol{S}_e$ the decoupled equations

$$\mathbf{K}[\boldsymbol{aH}] - \frac{h^2}{z}\mathbf{G}[\boldsymbol{tE}] = 0 \quad \text{and} \quad \mathbf{K}[\boldsymbol{aE}] + \frac{h^2}{y}\mathbf{G}[\boldsymbol{tH}] = 0$$

emerge.

As eqns.(11) and (12) are of substantially the same structure, numerical effort for their discretization is strongly reduced. As long as no symmetries are considered, the fundamental solution for the dynamic problem is given by

$$g(\boldsymbol{p},\boldsymbol{q}) = K_{B0}(jh\|\boldsymbol{p}-\boldsymbol{q}\|),$$

where K_{B0} denotes the modified Bessel function of the second kind and zero order. By separately evaluating the integrals involving regular and singular parts of the fundamental solution further economization is accomplished. This is due to the fact that the singular part, requiring most of the expense in numerical integration is independent of frequency and propagation constant and so must be computed only once.

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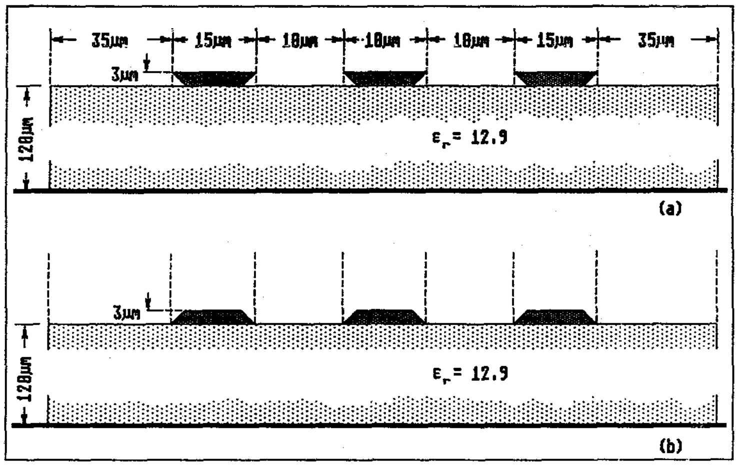


Fig.2: Open coplanar structure with non-rectangular conductor shape as due to underetching (a) and electrolytical growth (b).

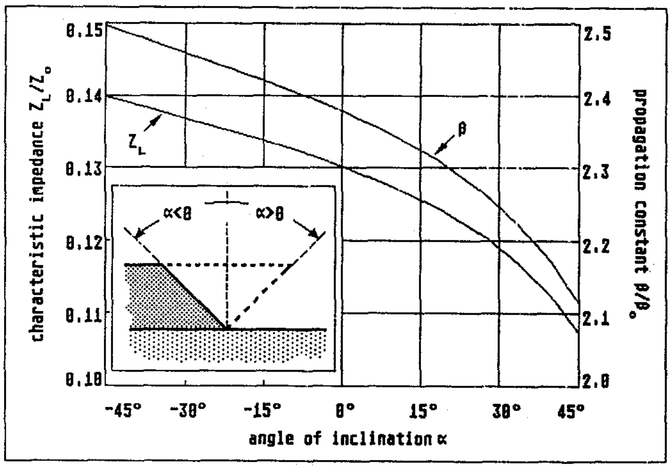
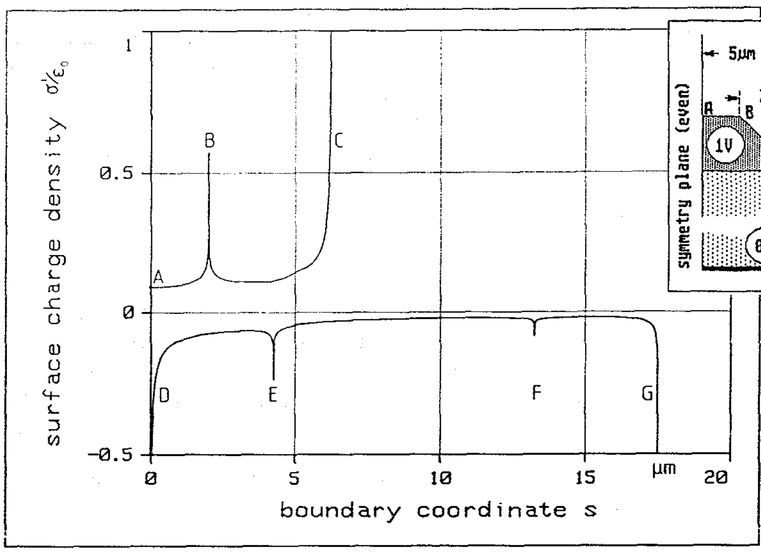


Fig.3: Influence of conductor shape on transmission line characterics for the structure shown in Fig.2.



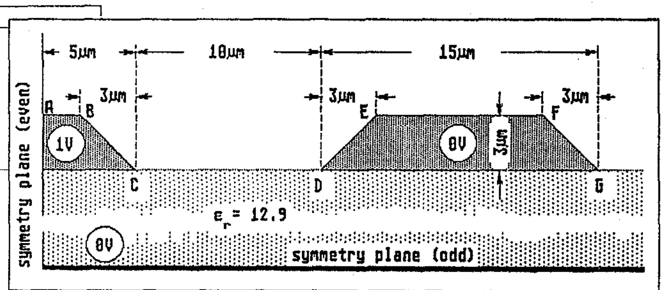


Fig.4: Normalized surface charge density σ/ε_0 on top and side walls of center and outer conductor for the structure of Fig.2(b) as computed with the static BIE formulation. Letters refer to the overlay.